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Graphentheorie

11. Übungsblatt WS 05/06
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Exercise 52

Let $d_1 \geq d_2 \geq \ldots \geq d_n$ be the degree sequence of G. Show that in an order $x_1, x_2, \ldots, x_n$, $d(x_i) = d_i$, the greedy algorithm uses at most

$$\max_i \min\{d_i + 1, i\}$$

colours, and so if k is the maximal natural number for which $k \leq d_k + 1$ then $\chi(G) \leq k$.

Exercise 53

Show that $\chi(G) + \chi(\overline{G}) \geq 2\sqrt{n}$.

Exercise 54

Let

$$p_G(x) = \sum_{i=0}^{n} (-1)^i a_i x^{n-i}$$

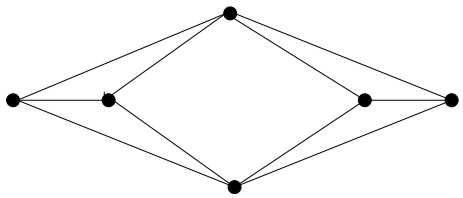
be the chromatic polynomial of G. Prove that

$$a_2 = \binom{m}{2} - k_3(G)$$

where $m = |E(G)|$ and $k_3(G)$ is the number of triangles in G.

Exercise 55

Calculate the chromatic polynomial of the following graph:



Exercise 56

Find the edge chromatic number of K_n.