

## Graphentheorie

7. Übungsblatt WS 05/06

Abgabetermin: 12.12.05

### Exercise 32

Let  $G(n, m)$  be a bipartite planar graph. Prove that  $m \leq 2n - 4$ .

### Exercise 33

Let  $G(n, m)$  be a bipartite planar graph. Prove that  $G$  contains a vertex of degree at most 3.

### Exercise 34

A mouse eats its way through a  $3 \times 3 \times 3$  cube of cheese by tunnelling through all of the  $27$   $1 \times 1 \times 1$  subcubes. If it starts at one corner and always moves on to an uneaten subcube, can it finish at the centre of the cube?

### Exercise 35

Show that it is impossible, using  $1 \times 2$  rectangles, to exactly cover an  $8 \times 8$  square from which two opposite  $1 \times 1$  corner squares have been removed.

### Exercise 36

Let  $d_1 \leq d_2 \leq \dots \leq d_n$  be a degree sequence of a planar graph. By making use of an upper bound for  $\sum d_i$ , show that if  $d_1 \geq 4$  then

$$\sum_{i=1}^n d_i^2 < 2(n+3)^2 - 62$$