Prof. Dr. Gabor Galambos
Dr. Marcus Oswald
Institut für Informatik
Universität Heidelberg

Graphentheorie

8. Übungsblatt WS 05/06
Abgabetermin: 16.01.06

Exercise 37

A line of a matrix is a row or a column of the matrix. Show that the minimum number of lines containing all 1's of a $(0,1)$-matrix is equal to the maximum number of 1's, no two of which are in the same line.

Exercise 38

Describe how the Hungarian method can be used to find a maximum matching in a bipartite graph.

Exercise 39

Two people play a game on a graph G by alternately selecting distinct vertices $v_0, v_1, \ldots$ such that, for $i > 1$, v_i is adjacent to v_{i-1}. The last player able to select a vertex wins. Show that the first player has a winning strategy iff G has no perfect matching.

Exercise 40

A diagonal of an $n \times n$ matrix is a set of n entries no two of which belong to the same row or the same column. The weight of a diagonal is the sum of the entries in it. Find a minimum-weight diagonal in the following matrix:

$$\begin{pmatrix} 4 & 5 & 8 & 10 \\ 7 & 6 & 5 & 7 \\ 8 & 5 & 12 & 9 \\ 6 & 6 & 13 & 10 \end{pmatrix}$$

Exercise 41

We call a component of a disconnected graph as odd (even) if it has odd (even) number of vertices. For a given graph G we denote the number of odd components by $q(G)$. Show that a tree G has a perfect matching iff $q(G - v) = 1$ for all $v \in V(G)$.